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REFLECTION OF ELECTROMAGNETIC WAVES FROM SOUND WAVES

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By

H. J. Schmitt

August 10, 1959

Technical Report No. 310

Cruft Laboratory

Harvard University

Cambridge, Massachusetts

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TR 310

Reflection of Electromagnetic Waves from Sound Waves

by

H. J. Schmitt

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Abstract

The reflection of electromagnetic waves normally incident on the wavefronts of a semi-infinite standing sound wave is discussed. By analogy with the Bragg reflection in optics, a maximum reflection occurs when the wavelength of the electromagnetic radiation in the sound perturbed region is twice the acoustic wavelength. Since the reflecting planes of maximum sound pressure disappear periodically, the reflected electromagnetic signal is modulated with the sound frequency. An experiment is described in which the Bragg reflection of 3 cm electromagnetic waves from a standing sound wave beneath a water surface is observed.

Introduction

With the advent of ultrasonic techniques numerous investigations have been made on the transmission and diffraction of light by ultrasonic waves [1]. Basically a sound disturbance produces a local variation of density and temperature in the medium of propagation. This in turn gives rise to a variation in the complex refractive index of the material and thus influences the phase, direction, frequency and amplitude of an electromagnetic wave traversing the sound-perturbed medium.

In the original Debye-Sears experiment [2], Fig. 1, a plane acoustic wave is approximated inside a liquid filled cuvette, and light is incident in a direction parallel to the acoustic wavefronts. The optical path length for light rays traversing regions of high acoustic pressure is larger than for rays in regions of pressure minima, so that at any instant of time the light wave has a certain phase distribution when leaving the cuvette. In this geometry the acoustic wave acts essentially as a diffracting "phase grating," except that the grating is constantly moving with the velocity of sound. Hence, in all diffracted orders, a frequency shift of the light wave is observed due to the Doppler effect [3]. If the experiment is carried out with a standing acoustic wave the diffraction grating disappears periodically so that an amplitude modulation of the transmitted light also occurs. It was observed later [4,5], that for slightly oblique incident light a pattern which can be interpreted as a selective Bragg reflection from the acoustic wavefronts is superimposed on the normal diffraction pattern, Fig. 2. Roughly, the electromagnetic radiation has a wavelength perpendicular to the sound wavefronts of $\frac{\lambda_{light}}{\sin\phi}$ which for small angles of incidence ϕ and very high acoustic frequencies can be comparable to the distance of the pressure maxima in the sound wave. In this case the addition of phase of waves reflected from subsequent "layers" can occur and give a substantial overall reflection, even though the variation of the refractive index may be extremely small, as is the case in all media for reasonable sound intensities*.

If the wavelength of the electromagnetic radiation is increased to about the microwave range or the UHF range, acoustic wavelengths and electromagnetic wavelength inside the medium of propagation can be made of the same order of magnitude even for low ultrasonic or sonic frequencies. Because of the wide choice of the ratio of acoustic to electric wavelength, the Bragg reflection can be made to occur at any desired angle of incidence. The particularly simple case of reflection of short electromagnetic waves normally incident on the plane of the sound wavefronts is treated in this report. Here the Bragg reflection of electromagnetic waves from sound waves is undisturbed by diffraction phenomena and can easily be observed experimentally. If the sound generator is in a liquid and directed toward the interface between liquid and air, a standing sound wave is produced owing to the large difference in the acoustic wave impedances of the media. An electromagnetic wave incident from

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*The propagation of light in the sound perturbed medium can be thought of as a zig-zag reflection under Bragg's angle, similar to the propagation of electromagnetic waves in hollow waveguides.

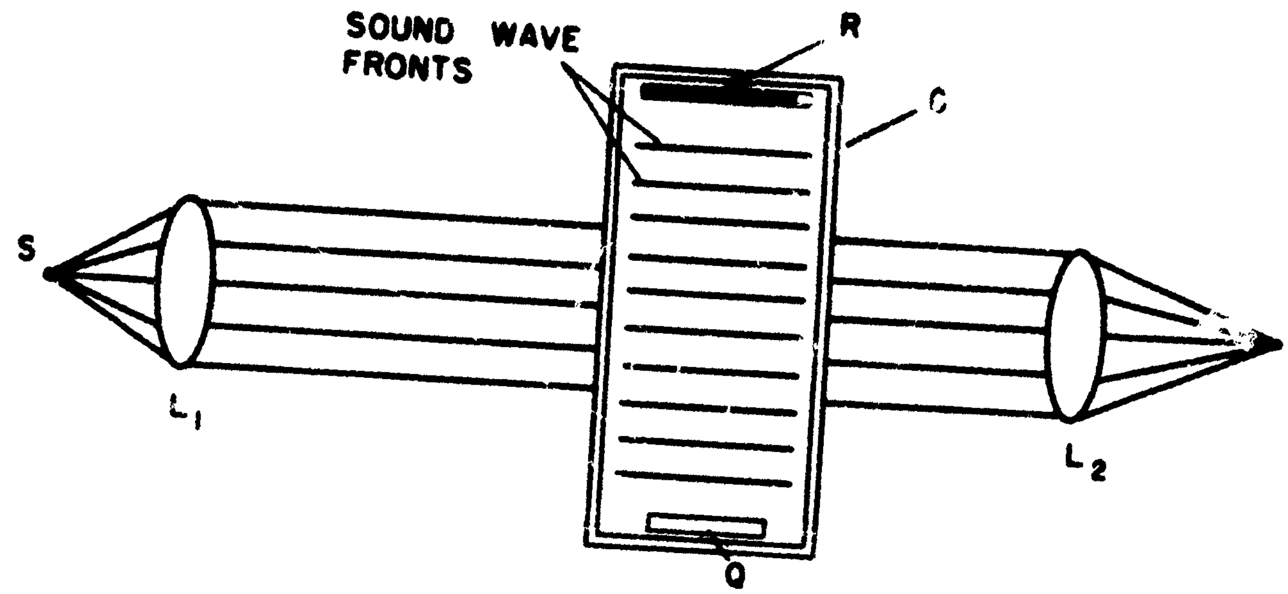

Fig. 1. Optical arrangement for diffraction of light by ultrasonic waves. L_1 and L_2 lenses, S light source, Q acoustic source, R reflector, C cuvette.

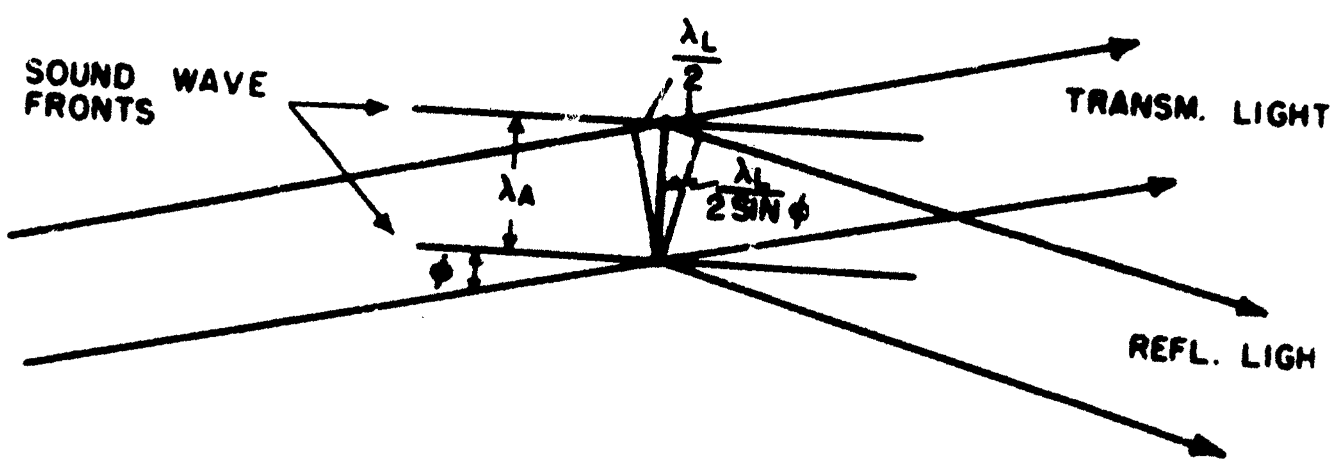

Fig. 2. Bragg reflection of light from sound waves.

the air is partially reflected at the interface where the electric wave impedance is discontinuous and partially reflected in the form of Bragg reflection from the standing sound wave. Since the pressure maximum in which Bragg reflection occurs disappears periodically, this part of the reflections appears as a modulation of the reflected electromagnetic signal.

Theory

The geometrical arrangement considered is sketched in Fig. 3. At $z = 0$ is a plane interface between a liquid or any other acoustically hard material (solid) and an acoustically soft material like air. A sound wave of frequency $f_A = \omega_A/2\pi$ and wavelength $\lambda_A = 2\pi/K_A$ propagating in the $-z$ direction is assumed to be ideally reflected at the boundary and to set up a standing acoustic wave in the liquid. The liquid is assumed to have a vanishing small acoustic absorption so that the instantaneous distribution of the sound pressure is given by

$$P = 2 P_o \sin K_A z \sin \omega_A t \tag{1}$$

Consequently the instantaneous distribution of the dielectric constant in the liquid medium is

$$\epsilon_2(z,t) = \epsilon_o(\epsilon_{r2} + \Delta\epsilon_{r2} \sin K_A z \sin \omega_A t) \tag{2}$$

where the dielectric constant of the unperturbed medium ϵ_{r2} and the small perturbation due to the sound wave $\Delta\epsilon_{r2}$ can both be complex, owing to possible absorption of the electromagnetic waves. The relation between $\Delta\epsilon_{r2}$ and the sound pressure itself poses a separate problem which is discussed later. It is sufficient here to note, that $\left|\frac{\Delta\epsilon_{r2}}{\epsilon_{r2}}\right|$ is of the order of 10^{-4} for even the highest sound intensities.

From the region $z < 0$ with a real dielectric constant ϵ_{r1} a plane electromagnetic wave of frequency $f = \omega/2\pi$ and free space wavelength $\lambda_o = 2\pi/\beta_o$

$$E_x^{inc.} = E_o e^{j(\omega t - \beta_o\sqrt{\epsilon_{r1}} \cdot z)}$$

impinges normally on the interface. The part of the electromagnetic signal reflected from the liquid medium will be computed by solving the wave equation in the slightly inhomogeneous medium 2 and subsequently satisfying the boundary conditions at $z = 0$.

From Maxwell's equations in a charge free liquid medium,

$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}$$

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} = \frac{\partial \epsilon \vec{E}}{\partial t}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \cdot \vec{D} = \epsilon \nabla \cdot \vec{E} + \vec{E} \cdot \nabla \epsilon = 0 \tag{3}$$

The wave equation may be derived in the following form:

$$-\nabla^2 \vec{E} + \nabla \frac{\vec{E} \cdot \nabla \epsilon}{\epsilon} = -\mu \frac{\partial^2 (\epsilon \vec{E})}{\partial t^2} \tag{4}$$

With the analysis restricted to a linearly polarized wave $\vec{E} = \hat{x} E_x$ and $\vec{H} = \hat{y} H_y$ it is seen that*

$$\vec{E} \cdot \nabla \epsilon = 0 \tag{5}$$

In the absence of the sound perturbation the electric field in the liquid has a harmonic time dependence of the form $e^{j\omega t}$. If the acoustic perturbation is present the electric field must show in addition a periodicity according to the sound frequency, so that in general

$$E_x = E(z, \omega_A t)\, e^{j\omega t} \tag{6}$$

Since, however, in all cases considered the frequency of the acoustic wave is much smaller than the frequency of the electromagnetic wave, $\omega_A/\omega \ll 1$, both $\epsilon = \epsilon(\omega_A t)$ and $E(z, \omega_A t)$ are slowly varying functions with respect to time compared to the term $e^{j\omega t}$. They can be considered constants under the differential on the right side of (4). The scalar wave equation in the liquid region is thus simplified to

$$\frac{\partial^2 E(z, \omega_A t)}{\partial z^2} + (K^2 + \Delta K^2 \sin K_A z)\, E(z, \omega_A t) = 0 \tag{7}$$

where $K^2 = \omega^2 \mu \epsilon_o \epsilon_{r2}$ and $\Delta K^2 = \omega^2 \mu \epsilon_o \Delta \epsilon_{r2} \sin \omega_A t$

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*This term does not drop out for oblique incidence, when $\vec{E}$ lies in the plane of incidence.

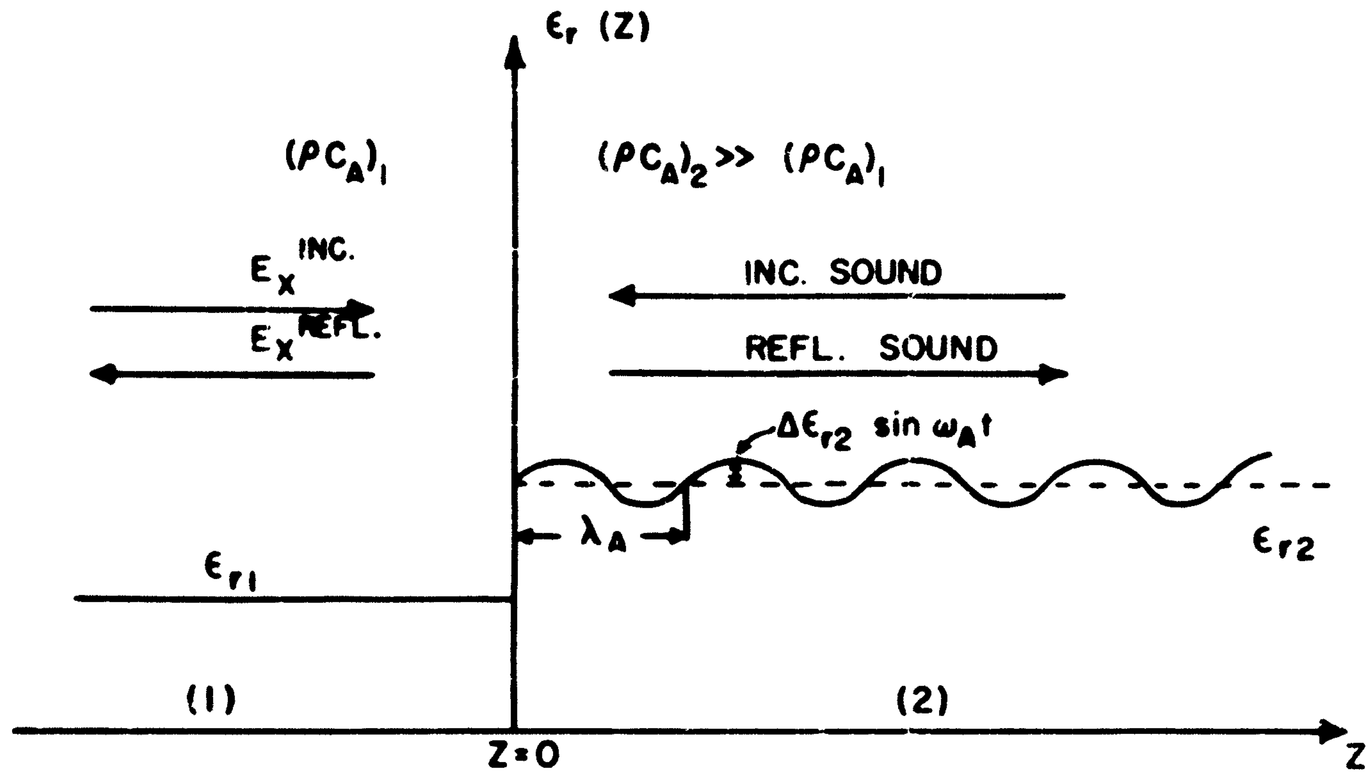

Fig. 3. Geometrical arrangement for reflection of electromagnetic waves from sound wave. (g = density, c_A = sound velocity)

The solution of this equation will be carried out by a first-order perturbation method, which is based on the fact that the perturbation term in (7) $\Delta K^2 \sin K_A z$ is a very small quantity compared to the square of the propagation constant in the unperturbed medium K^2:

$$\frac{\Delta K^2}{K^2} = \frac{\Delta\epsilon_{r2}}{\epsilon_{r2}} \sin \omega_A t \ll 1$$

The solution of the wave equation is set up in the form

$$E(z, \omega_A t) = E_t e^{-jKz} \left[1 + g(z, \omega_A t) \right] \tag{8}$$

where $g(z,\omega_A t)$ denotes the perturbation in the electric field due to the sound wave. The insertion of (8) in (7) yields the following differential equation for $g(z,\omega_A t)$:

$$g'' - 2jKg' = -\Delta K^2 \sin K_A z \,(1 + g) \tag{9}$$

Here the perturbation term $g(z,\omega_A t)$ may be neglected in a first-order approximation on the right side as long as $g(z,\omega_A t) \ll 1$. The remaining second order differential equation is solved, in the stationary case, by

$$g(z,\omega_A t) = \frac{\Delta K^2}{4K^2 - K_A^2}\left(\frac{2jK}{K_A} \cos K_A z - \sin K_A z\right) \tag{10}$$

The final expression for the electric field in the sound-perturbed medium is obtained by combining (10) with (8) and (6).

$$\vec{E} = \hat{x}\, E_t e^{j(\omega t - Kz)} \left[1 + \frac{\Delta K^2}{4K^2 - K_A^2}\left(\frac{2jK}{K_A} \cos K_A z - \sin K_A z\right)\right] \tag{11}$$

This solution is valid for all K except for values in the immediate vicinity of $4K^2 - K_A^{\,2} = 0$, for which the wavelength of the electric field in the medium is exactly twice the acoustic wavelength. Here the condition that $g(z,\omega_A t)$ must be small compared to unity is not fulfilled and the electric field apparently blows up.* It should be noted that this can only occur if the liquid medium is lossless for electromagnetic waves. In view of the very small value of $\left|\frac{\Delta K}{K}\right|^2$ even extremely low electric loss tangents of the liquid medium in the order of $\tan\delta \sim 10^{-4}$, as always encountered in practice, will

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* This infinity appears also in the theory of traveling wave tubes, when small periodic inhomogeneities on the delay line are considered.

prevent the singularity. The magnetic field of the electromagnetic wave in the liquid medium is obtained from (3), again with the understanding that $\omega_A \ll \omega$,

$$\vec{H} = \hat{y}\,\frac{j}{\mu\omega}\,\frac{\partial E(z,\omega_A t)}{\partial z}\,e^{j\omega t} = \hat{y}\,\frac{E_t}{Z_2}e^{j(\omega t - Kz)}\left[1 + \frac{\Delta K^2}{4K^2 - K_A^2}\left(\sin K_A z + j\frac{2K^2 - K_A^2}{K_A K}\cos K_A z\right)\right] \tag{12}$$

where Z_2 is the wave impedance in the unperturbed medium $Z_2 = \frac{\mu\omega}{K}$. The distribution of the field in the sound-perturbed medium is that of a propagating wave, modulated in space with a periodicity given by the acoustic wavelength.

At the boundary between the two semi-infinite media, at $z = 0$*, the condition of continuity of the tangential electric and the tangential magnetic fields have to be fulfilled. The following amplitude relation for the electric field strength of the reflected wave $E_x^r = E_r\,e^{j(\omega t + \beta_o \sqrt{\epsilon_{r1}}\, z)}$ is obtained

$$E_o + E_r = E_t\left[1 + 2\eta \sin \omega_A t\right]$$

$$E_o - E_r = \frac{Z_1}{Z_2}E_t\left[1 + \frac{2K^2 - K_A^2}{K^2}\,\eta \sin \omega_A t\right];\ \left(Z_1 + \sqrt{\frac{\mu_o}{\epsilon_o \epsilon_{r1}}}\right) \tag{13}$$

where the abbreviation is used

$$\eta = j\,\frac{K}{K_A}\,\frac{\Delta K^2/\sin \omega_A t}{4K^2 - K_A^2} = j\,\frac{\Delta\epsilon_{r2}}{\epsilon_{r2}}\,\frac{K/K_A}{4 - (K/K_A)^2} \tag{14}$$

The reflection factor is obtained from (13) as follows:

$$r = \frac{E_r}{E_o} = \frac{\left(\frac{Z_2}{Z_1} - 1\right) + \eta\left[2\frac{Z_2}{Z_1} - \left(2 - \left(\frac{K_A}{K}\right)^2\right)\right]\sin \omega_A t}{\left(\frac{Z_2}{Z_1} + 1\right) + \eta\left[2\frac{Z_2}{Z_1} + \left(2 - \left(\frac{K_A}{K}\right)^2\right)\right]\sin \omega_A t} \tag{15}$$

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*The effect of possible surface movements is discussed in a later section.

Finally, since η is a small quantity whenever (11) and (12) are valid, the reflection factor may be expressed by the leading terms of a Taylor series in η,

$$r = \frac{\left(\frac{Z_2}{Z_1} - 1\right)}{\left(\frac{Z_2}{Z_1} + 1\right)} + \frac{2\,\frac{Z_2}{Z_1}\left(\frac{K_A}{K}\right)^2}{\left(\frac{Z_2}{Z_1} + 1\right)^2}\,\eta \sin \omega_A t \tag{16}$$

It is seen that the reflection factor reduces to the usual expression for the reflection of plane electromagnetic waves at the interface of medium 1 and the unperturbed medium 2 if $\eta = \frac{\Delta\epsilon_{r2}}{\epsilon_{r2}} = 0$: $r(\eta = 0) = |r_o|\, e^{j\phi} = \left(\frac{Z_2}{Z_1} - 1\right)\Big/\left(\frac{Z_2}{Z_1} + 1\right)$. The presence of the sound wave is expressed in the second term which is due to a type of Bragg reflection from the equidistant plane "layers" of increased refractive index in the liquid medium.

Since the sound grating disappears periodically the reflected wave has the well-known form of a weakly modulated signal. The amplitude may be written

$$E_r = E_o\,(r_o + M \sin \omega t) = E_o\, r_o\,(1 + m \sin \omega_A t) \tag{17}$$

where

$$m = \frac{M}{r_o} = \frac{|M|\, e^{j\psi}}{|r|\, e^{j\phi}} = \frac{2\,\frac{Z_2}{Z_1}\left(\frac{K_A}{K}\right)^2 \delta}{\left(\frac{Z_2}{Z_1}\right)^2 - 1} \tag{18}$$

is the modulation index. It is proportional to $\left|\frac{\Delta\epsilon_{r2}}{\epsilon_{r2}}\right|$ and therefore extremely small. In the particular case in which ϵ_{r2} is real, the modulation index is purely imaginary and, in a first approximation, the reflected signal may be considered to be phase modulated*. In the general case, for a lossy medium, m is complex and the reflected signal is modulated in phase and amplitude. From a practical

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*If desired, the phase modulation may be converted into amplitude modulation by shifting the carrier 90 degrees in phase.

point of view the field strength in one of the two side bands of the modulated signal is interesting. In an experimental investigation the reflected wave can be received in a superheterodyne system and the field strength in the side bands (i.e., at angular frequencies $\omega + \omega_A$ or $\omega - \omega_A$) detected directly. Its intensity can be used as a quantitative indication for the Bragg reflection. The instantaneous value of the electric field strength in the reflected wave at a fixed point in space Kz = const. = ζ is obtained from (17).

$$(E_r)_{inst.} = R_e \left\{ E_o \, |r_o| \, e^{j(\omega t + \zeta + \phi)} + E_o \, |M| \sin \omega_A t \, e^{j(\omega t + \zeta + \psi)} \right\} \tag{19}$$

The first term in (19), due to normal interface reflection, constitutes the carrier. The second term, due to internal Bragg reflection, represents the side bands. The normalized field strength in the side band is

$$\left| \frac{E_r^s}{E_o} \right| = \frac{|M|}{2} = \left| \frac{\frac{Z_2}{Z_1} \; \frac{K_A}{K} \; \frac{\Delta\epsilon_{r2}}{\epsilon_{r2}}}{(\frac{Z_2}{Z_1} + 1)^2 \; [4 - (\frac{K_A}{K})^2]} \right| \tag{20}$$

Discussion

The reflection of the electromagnetic signal from the standing sound wave is mainly a function of the ratio of the propagation constant of the electromagnetic wave to the propagation constant of the acoustic wave, or roughly, a function of the ratio of wavelengths. For a discussion of this behavior and its dependence on the loss tangent of the liquid medium, that is, on the penetration depth of the electromagnetic wave into the sound perturbed medium, two particular cases have been investigated in more detail. In both cases it is assumed that the electromagnetic wave impinges from air ($\epsilon_{r1} = 1$) on a medium characterized by a complex dielectric constant

$$\epsilon_{r2} = \epsilon' - j\epsilon'' = \epsilon' (1 - j \tan \delta)$$

In the first case a value of $\epsilon' = 2$, roughly corresponding to oil, is assumed and different loss tangents for the electromagnetic radiation ($\tan\delta = 0$; 0.01; 0.1; and 1.0) are chosen. The response due to Bragg reflection has been computed as a function of $\frac{\lambda_o}{\lambda_A}$ and is plotted in terms of the relative variation in the dielectric properties $\left|\frac{\Delta\epsilon_{r2}}{\epsilon_{r2}}\right|$ in Fig. 4. It is well known from optics that a Bragg reflection occurs, whenever the distance between subsequent reflecting layers is equal to one half of the wavelength of the incident light. It is seen in Fig. 4 that the maximum of the modulated part of the reflected signal lies, for small loss tangents, at $\frac{\lambda_o}{\lambda_A} = 2.82$, i.e., the electric wavelength in the medium $\sim \frac{\lambda_o}{\sqrt{\epsilon'}} = \frac{\lambda_o}{\sqrt{2}}$ is exactly twice the acoustic wavelength, corresponding to the usual optical Bragg condition. However, and differing from the selective reflection at subsequent planes, a Bragg reflection also occurs if this condition is not exactly fulfilled, although with smaller amplitude. A similar effect with reference to the Bragg reflection as a function of the angle of incidence of light on acoustic waves has been pointed out by Extermann et al. [5]. It is attributed to the steadily varying distribution of the refractive index in the sound wave. Also, owing to the sinusoidal variation of the dielectric constant, no higher orders of the Bragg reflection occur if the distance of sound maxima is a multiple of half the wavelength of the electromagnetic wave. In optics Bragg reflection is assumed to occur from thin reflecting planes. In this case the perburbation function $\Delta K^2 \sin K_A z$ on the right side of (9) would be replaced by the Fourier representation of a pulse function

$$\Delta K^2 \sum_{n=1,3,5\ldots}^{\infty} C_n \sin(n K_A z)$$

which leads to the correct Bragg relation for the particular wavelength ratios at which the selective reflection occurs

$$2\lambda_A = n\lambda_{light} \quad \text{(for normal incidence.)}$$

For the sinusoidal distribution of the dielectric constant, the homogeneous decrease of the Bragg-reflection with decreasing ratio $\frac{\lambda_o}{\lambda_A}$ can be made plausible by the fact, that the gradient of the variation in dielectric properties decreases steadily in terms of the wavelength of radiation. If the electric wavelength is made large compared to the acoustic wavelength, the Bragg reflection decreases because the perturbed medium becomes more and more homogeneous in character.

It is seen that the maximum reflection also decreases steadily with an increasing loss tangent of the medium 2. This can be explained in terms of the penetration depth, since for higher loss tangents fewer "layers" of maximum acoustic pressure take part in the Bragg reflection. However, even for the extremely high loss tangent $\tan\delta = 1$, there is still an indication of a weak maximum for the reflected signal. For a lossless medium, $\tan\delta = 0$, the perturbation method is not valid in the immediate vicinity of the peak (actually infinity). With $\left|\frac{\Delta\epsilon_{r2}}{\epsilon_{r2}}\right| = 10^{-4}$ the condition $g(z,\omega_A t) \ll 1$ is still satisfied within the range of this drawing, and the response should be accurate as far as plotted. The infinity is due to the unphysical assumption of an infinite number of subsequent regions of extreme pressure that contribute to the build up of the Bragg reflection.

In the second case a situation easily achieved in practice is considered. The standing sound wave is set up below a water-air surface. To permit a later comparison with experiment, the dielectric properties of water in the microwave range are used for the computation, and it is assumed that the variation in $\frac{\lambda_o}{\lambda_A}$ is achieved by changing the acoustic frequency*. The dielectric properties of fresh water have been determined

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*If the frequency of the electromagnetic wave is changed it has to be noted that the dielectric properties of water change with frequency, showing the typical behavior of a relaxation process.

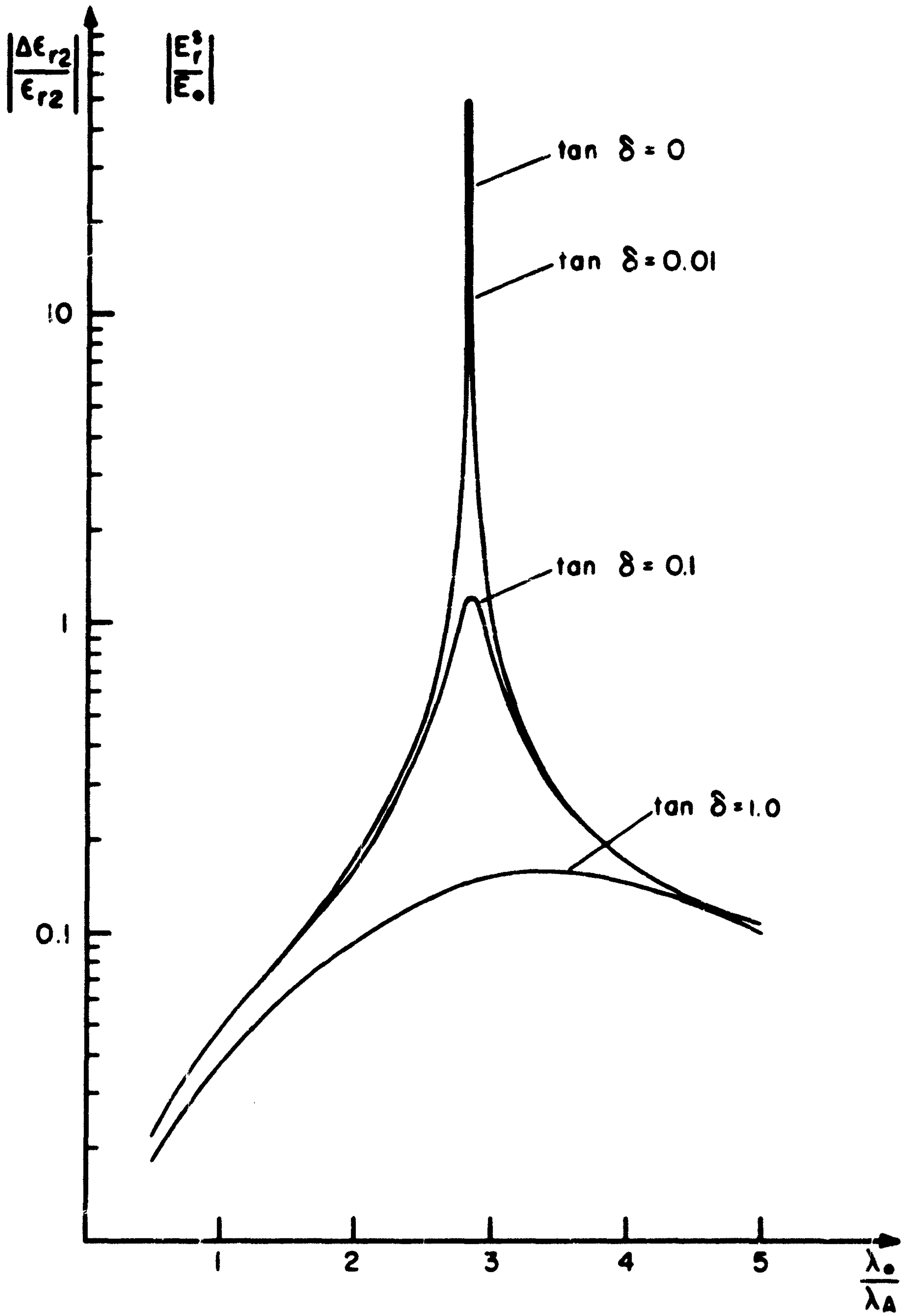

Fig. 4. Field strength in side band of modulated signal reflected from sound wave over wavelength ratio. ($\epsilon_{r2} = 2\,(1 - j\tan\delta)$, $\epsilon_{r1} = 1$)

by Saxton [6]. At around λ_o = 3cm (X-band) the measured values are approximately

$$\epsilon_{r2} = 60 - j\,30$$

These values depend only very little on salt concentration since the electrolytic conduction by ions is practically zero at microwave frequencies. The computed response is plotted in Fig. 5 again as a function of free space electric wavelength over acoustic wavelength. It shows essentially the same maximum if the Bragg condition $\frac{\lambda_o}{\sqrt{\epsilon'}} \sim 2\lambda_A$ is fulfilled. The relative field strength in the sideband is somewhat less than for a medium with a smaller real part of the dielectric constant and a comparable loss tangent, owing to the smaller ratio of wave impedances Z_2/Z_1 in Eq. 20. Note, however, that the ordinate contains the relative variation of the dielectric constant which is still undetermined and is different for different materials, for a given sound pressure.

The question, whether the Bragg reflection can be observed experimentally, depends entirely on the change in the dielectric properties of the liquid produced by the sound pressure. Let R denote the specific refraction of the liquid medium. Then the Lorentz-Lorenz relation states that

$$R = \frac{\epsilon_{r2} - 1}{\epsilon_{r2} + 2} \; \frac{1}{\rho} \tag{21}$$

It is known, [7], that R is a constant for any given wavelength of electromagnetic radiation and widely independent of the density ρ, the state of the material and the chemical binding. Hence, for a varying density,

$$\Delta\epsilon_{r2} = \frac{\Delta\rho}{\rho} \; \frac{(\epsilon_{r2} - 1)(\epsilon_{r2} + 2)}{3 - (\epsilon_{r2} - 1)\,\frac{\Delta\rho}{\rho}} \tag{22}$$

Since the relative variation in density is always small in liquids and solids, the term $(\epsilon_{r2} - 1)\frac{\Delta\rho}{\rho}$ may be neglected in the denominator in order to linearise the equation. For gases, where $\frac{\Delta\rho}{\rho}$ may be large, the difference $(\epsilon_{r2} - 1)$ is always small, so that this step is also justified in the gaseous state. The adiabatic change of the density of the medium due to acoustic pressure may be computed directly from the sound velocity c_A

$$c_A^2 = \left(\frac{\partial p}{\partial \rho}\right)_{ad.} = \left(\frac{\Delta p}{\Delta \rho}\right)_{ad.} \tag{23}$$

For the relative variation of the dielectric constant one obtains with (23)

$$\left|\frac{\Delta \epsilon_{r2}}{\epsilon_{r2}}\right| = \frac{\Delta p}{\rho c_A^2} \left|\frac{(\epsilon_{r2} - 1)(\epsilon_{r2} + 2)}{3\epsilon_{r2}}\right| \tag{24}$$

where Δp is the amplitude of the acoustic pressure in a pressure maximum of the standing wave*. It should be noted, that in this relation ϵ_{r2} is assumed to be independent of temperature. The relation takes into account only the effect of adiabatic compression, not, however, eventual effects of rising temperature in an adiabatic compression on the dielectric properties. Such effects may be superimposed and must be treated in a separate way. For water the temperature dependence of ϵ_{r2} is small, however, at microwave frequencies.

For the reflection of microwaves from a sound wave ($\epsilon' = 60$; $\epsilon'' = 30$; at $\lambda_o = 3$cm) a numerical estimate yields for the relative change of ϵ_{r2}

$$\left|\frac{\Delta \epsilon_{r2}}{\epsilon_{r2}}\right| = 31.2 \cdot 10^{-4} \quad \left(\rho = 10^3 \frac{Kg}{m^3},\ c_A = 1538 \frac{m}{sec} \right)$$

if an acoustic pressure of 1/3 atmosphere = $9.81/3 \times 10^4$ Kg/m sec. is assumed. With this value the field strength in the sideband of the modulated signal, in the maximum of the Bragg reflection, is approximately

$$\left|\frac{E_r^s}{E_o}\right| = 0.1 \left|\frac{\Delta \epsilon_{r2}}{\epsilon_{r2}}\right| = 3.12 \cdot 10^{-4}$$

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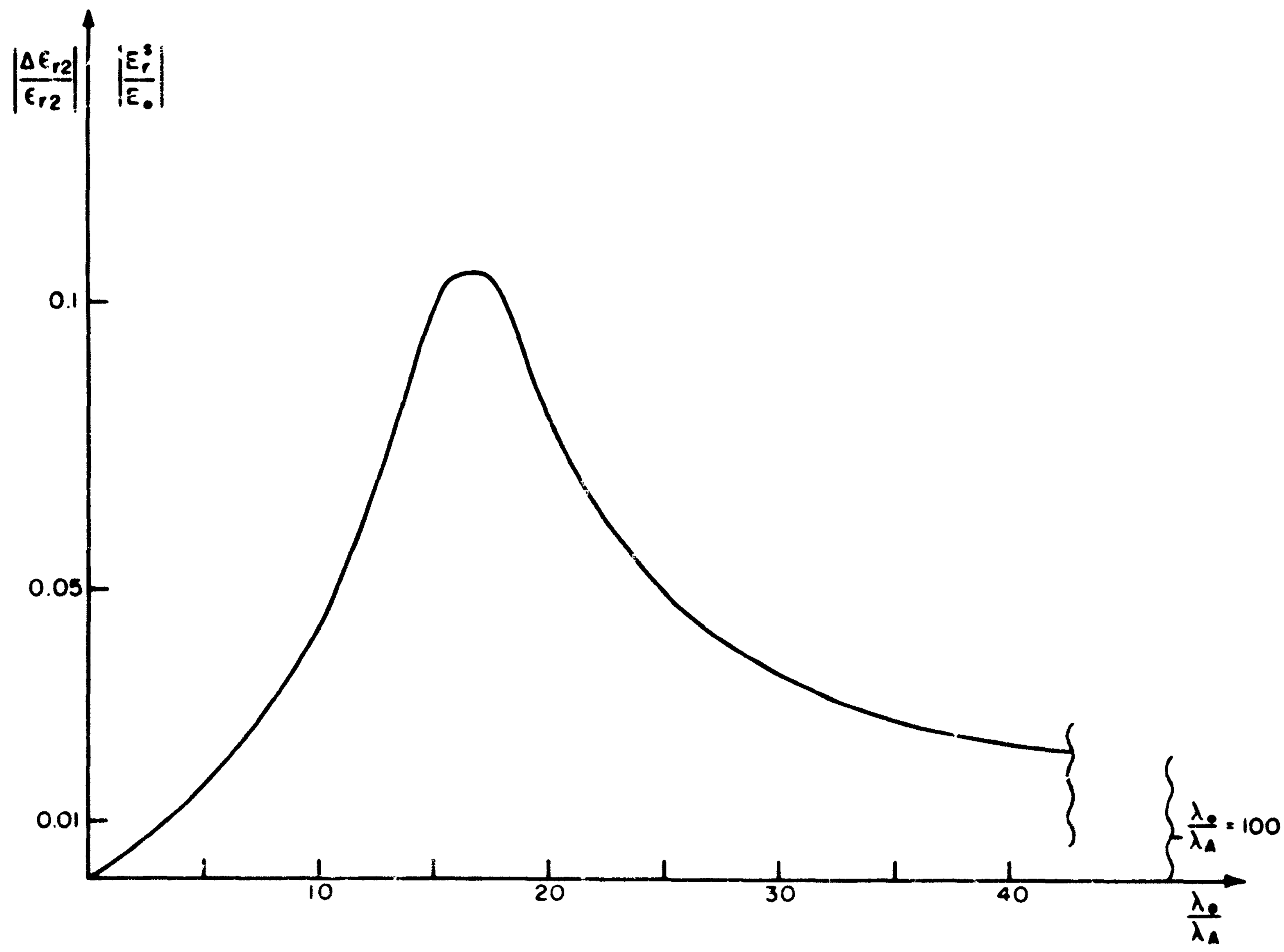
*A different way has been used by Fox et al. [8]. They derived the change in conductivity of salt solutions due to an adiabatic sound compression by comparing the empiric values of the isothermic dependence of the conductivity on pressure at two different temperatures. With a separate computation of the temperature rise in the adiabatic compression, the temperature effect could be added to the effect of isothermic compression.



$\left|\frac{\Delta\epsilon_{r2}}{\epsilon_{r2}}\right|$ $\left|\frac{E_r^s}{E_o}\right|$
0.1
0.05
0.01
10
20
30
40
$\frac{\lambda_o}{\lambda_A} = 100$
$\frac{\lambda_o}{\lambda_A}$

That is, the intensity in the sideband is about 70db below the intensity of the incident wave. This should pose no serious difficulty in an experimental observation*. The corresponding value in a fairly lossy oil with $\epsilon' = 2$ and $\tan\delta = 0.01$ is

$$\left|\frac{E_r^s}{E_o}\right| = 11.5 \left|\frac{\Delta\epsilon_{r2}}{\epsilon_{r2}}\right| = 1.49 \cdot 10^{-4} \qquad \begin{pmatrix} c_A = 1420 \frac{m}{sec} \\ \rho = 0.835\ 10^3 \frac{Kg}{m^3} \end{pmatrix}$$

It should be noted, however, that this analysis assumes perfectly plane acoustic waves as well as plane electromagnetic waves. These conditions are never fulfilled in experiments and can be a serious limitation on the detectability of the reflection. Furthermore, no acoustic losses have been assumed, but this is probably of lesser importance.

Experiments

In order to demonstrate the Bragg reflection from sound waves, a preliminary experiment was carried out in which the back scattering of microwaves with a 3.2cm free space wavelength from a standing sound wave under the air-water surface was observed. Although, with an acoustic frequency of 60 Kc/sec for which equipment was readily available, the condition of maximum Bragg reflection was far from fulfilled, a modulation of the reflected electromagnetic signal could still be detected.

The system used is schematically sketched in Fig. 6. A photograph is shown in Fig. 7 A steel watertank** with the dimensions $1.54 \times 2.13 \times 0.76\ m^3$ contains a magnetostrictive underwater sound transducer with a circular aperture of radius 8.3 cm. The transducer

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*Assuming an illumination of the water surface with 1 Watt/m^2, the intensity in the sideband of the reflected wave will be approximately 10^{-7} Watts/m^2. Even with a small absorption cross section of the receiving antenna of $10^{-2} m^2$ the received power is of the order of 10^{-9} Watts, which is well above the limit of standard superherterodyne receivers.

**Thanks are due to Professor F. Hunt and Dr. H. Flynn for providing the acoustic facilities in Cruft Laboratory.

has a resonance frequency around 60 kc/sec. It is fed by a 200 Watt radio frequency oscillator which can be either pulsed in various repetition rates or operated in c. w. The oscillator is continuously tunable in a wide frequency band covering the 60 kc/sec region. The transducer is directed against the water surface and produces a standing sound wave in which planes of pressure maxima, at any instant of time, are $\lambda_A = 2.56$cm apart. The depth of the water filled section is about 60 cm. It is chosen in such a way that the acoustic "resonator" formed by the water surface and the aperture of the transducer is not in antiresonance. Clearly, the water-air interface is not in the far zone region of the sound source, and thus the actual distribution of the acoustic field approximates a plane wave configuration only in a narrow region above the speaker. In addition, reflections from the side walls of the unlined tank may perturb the pressure distribution. The maximum acoustic pressure generated in the standing wave is estimated from

$$(\Delta p)_{atm.} = 2 \cdot 1.24 (F)^{\frac{1}{2}}$$

where F is the acoustic power in Watts per cm^2 [8]. With an efficiency in the order of 10% and 200 Watts electric power, the pressure is of the order of 0.75 atm, probably somewhat lower, so that the numerical values for $\left|\frac{\Delta\epsilon_{r2}}{\epsilon_{r2}}\right|$ used in the discussion of the theoretical results should apply approximately.

The electromagnetic system consists of a 2 resonator klystron (Varian X 2lb, $\lambda_o = 3.2$cm) with an estimated output of 2 Watts in the particular mode of operation. The c.w. microwave energy is fed through a variable attenuator and a tuner into a magic T (M in Fig. 6), where it is separated into two parts in the equal branches of the T. It is radiated from one branch by a rectangular horn antenna (aperture 9.5 x 15 cm^2), directed toward the water-air interface. The antenna is rigidly mounted at a height of about 1m above the water level and in line with the standing acoustic wave. It also serves to receive the modulated signal reflected from the water. In the second branch of the T a fixed

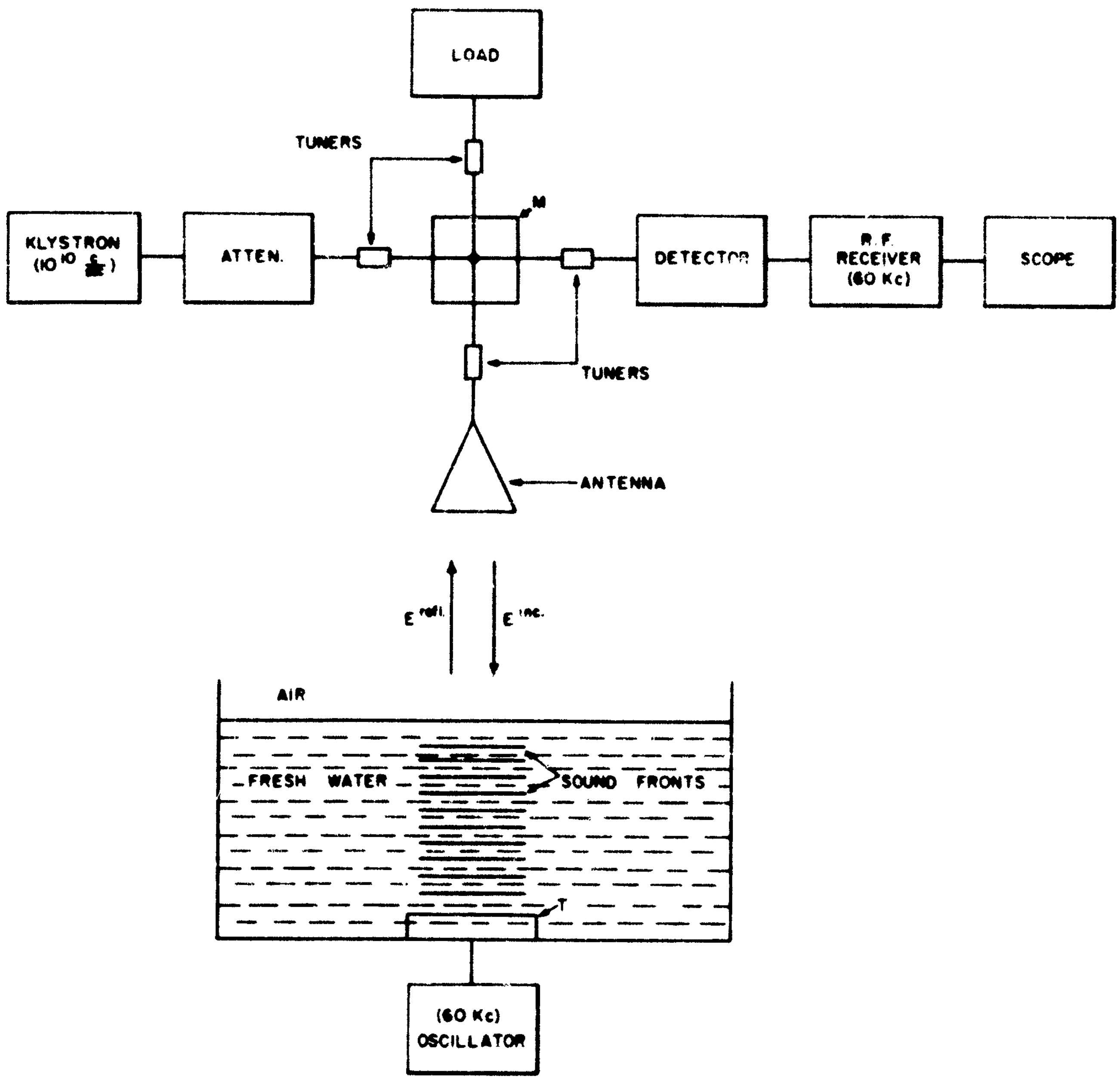

Fig. 6. Schematic diagram of experimental set up to detect modulation of electromagnetic waves by sound waves.

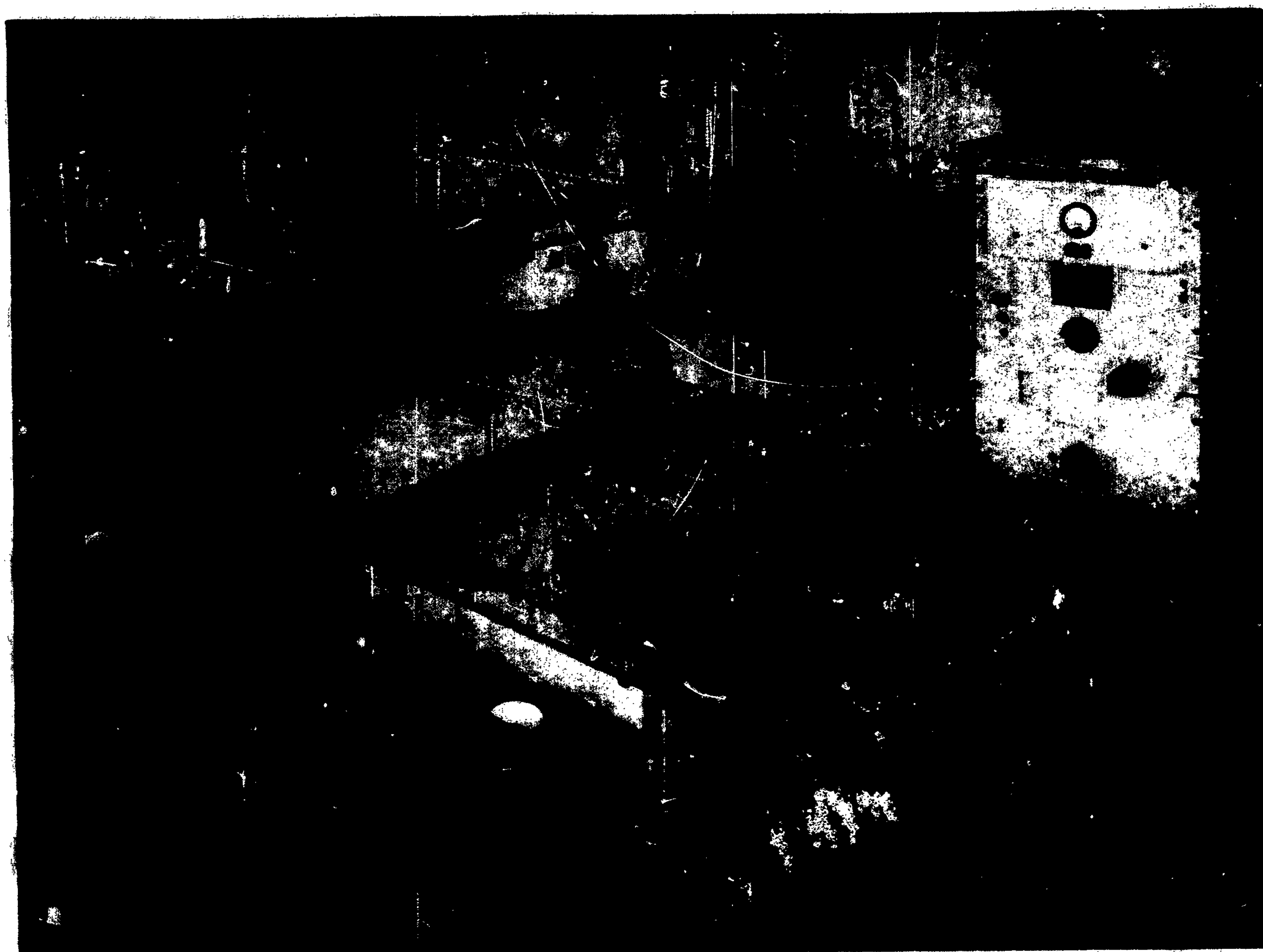

Fig. 7. Photograph of experimental set up.

waveguide load in connection with a sliding screw tuner provides for any desired amount and phase of reflection of that part of the microwave energy that is traveling into this side. The output branch of the T leads directly through a crystal demodulator into a tunable radio frequency receiver type CFT 46154. The receiver has a sensitivity of about 10^{-8} volts at 60 kc/sec for the minimum detectable signal.

In this arrangement the detecting system operates as a simple demodulator with a subsequent high amplification of the modulating signal. The only difference is that in view of the small modulation index the main part of the carrier frequency has to be cancelled out in order to avoid the overloading of the crystal. A further difficulty arises in the fact that the reflected electromagnetic signal is modulated in both amplitude and phase. In order to maximize the response after demodulation, it is necessary to convert the phase modulation into amplitude modulation. This can be achieved by adjusting the cancelling signal from the auxiliary arm of the magic T in such a way that the total carrier signal at the crystal detector has the proper phase relation. With (19) it follows that the instantaneous value of the total electric field at the crystal is

$$(E_r)_{inst.} = \text{const.}\ [\,|R|\cos(\omega t + \zeta + \vartheta) + |M|\sin\omega_A t\cos(\omega t + \zeta + \xi)\,] \quad (25$$

where $|R|$ and ϑ contain the contribution in amplitude and phase to the carrier frequency by the signal reflected from the interface and the signal reflected from the auxiliary branch of the T. If a square law is assumed for the detector, the response to the voltage across the detector at the modulation frequency is

$$u(\omega_A t) = \text{const.}\ |M|\ |R|\ \cos(\xi - \vartheta)\sin\omega_A t \quad (26$$

It is seen that the response is optimized if $\xi = \vartheta$, i. e., if (25) has the form of a purely amplitude-modulated signal. For relative measurements this detecting device is sufficient if, for example, the acoustic frequency is changed. An absolute calibration appears to be difficult and has not been attempted.

After a proper tuning of the cancelling signal and a careful check of other obvious disturbances, like electric cross coupling between power oscillator and receiver and microphonic effects in the klystron, the modulation of the reflected electromagnetic wave by the sound wave was clearly observed on the oscilloscope. In spite of the unfortunate ratio of electric and acoustic wavelength ($\frac{\lambda_o}{\lambda_A} = 1.25$, instead of the optimal ratio of 17, Fig. 5) the response on the scope was about 20db above the noise level, produced mainly by the klystron and in part by the receiver. The magnitude was the same for pulsed sound waves as for the steady standing wave. The effect could be disturbed by producing additional waves on the water surface, which would tend to change the proper cancellation of the carrier, and it would vanish completely, if the sound source was moved from the location directly underneath the horn antenna or tilted in any other direction than straight up, without changing the water surface.

The argument might be raised, that the modulation of the reflected electromagnetic signal could also occur in the form of a phase modulation from the microscopic movement of the surface. The amplitude with which water particles oscillate at the interface is given by

$$A = \frac{\Delta p}{\rho c\, \omega_A}$$

if ΔP is again the maximum sound pressure in the standing sound wave. This produces a phase modulation of the electromagnetic wave reflected directly from the interface. Neglecting the term due to Bragg reflection in this calculation, the reflected signal may be written in the form

$$E_x^r = E_o \frac{\frac{Z_2}{Z_1} - 1}{\frac{Z_2}{Z_1} + 1} e^{j(\omega t + \beta_o z + \frac{2\pi A}{\lambda_o} \sin \omega_A t)} = E_o |r_o| e^{j\phi} e^{j(\omega t + \beta_o z)} (1 + j \frac{2\pi A}{\lambda_o} \sin \omega_A t) \qquad (27)$$

It is seen that the reflected signal in this case also contains side bands at $\omega + \omega_A$ and $\omega - \omega_A$. The normalized field strength in a side band follows from (27)

$$\left| \frac{E_r^s}{E_o} \right|_{surface} = \frac{1}{2} \frac{2\pi A}{\lambda_o} |r_o| = \frac{\pi \Delta p}{\rho c \omega_A \lambda_o} |r_o| = \frac{|r_o|}{2} \frac{\Delta p}{\rho c_A^2} \frac{\lambda_A}{\lambda_o} \qquad (28)$$

Taking the wavelength ratio for maximum Bragg reflection from water ($\frac{\lambda_o}{\lambda_A}$ = 17), and assuming again ΔP = 1/3 atm. one obtains, with $|r_o|$ = 0.785,

$$\left|\frac{E_r^s}{E_o}\right|_{surface} = 3.2 \cdot 10^{-7}$$

which is about a factor of 10^3 smaller than the corresponding modulation due to the Bragg reflection. Since for a given wavelength ratio (28) is widely independent of the frequency, it can be concluded that the Bragg effect is always larger in water than the effect of surface movements, as long as one operates at a ratio close to the maximum of Bragg reflection*. Even in the situation encountered in the experimental test, where $\frac{\lambda_o}{\lambda_A}$ = 1.25, the sideband level due to surface movements

$$\left|\frac{E_r^s}{E_o}\right|_{surface} = 1.33 \cdot 10^{-10}\ \Delta p$$

is smaller by a factor of more than two than the corresponding value from the Bragg reflection

$$\left|\frac{E_r^s}{E_o}\right| = 2.96 \cdot 10^{-10}\ \Delta p$$

Thus the assumption that the response observed experimentally really originates from the interior of the liquid medium is justified.

In some earlier experiments a modulation was also obtained if an oil layer of about 1cm thickness was brought on top of the water surface. However, no comparison of the response from water and oil has been attempted yet.

Conclusions

Judging from the results of the approximate theory, as well as from the preliminary experiment, it appears to be relatively easy to demonstrate and measure the interaction of acoustic waves with electromagnetic waves through a type of Bragg reflection. Clearly, additional

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*At least for frequencies in the microwave range.

experiments are needed to verify the existence and shape of the predicted maximum of the effect of sound waves on the reflected signal.

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